MATH 4750 / MSSC 5750

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Probability Review

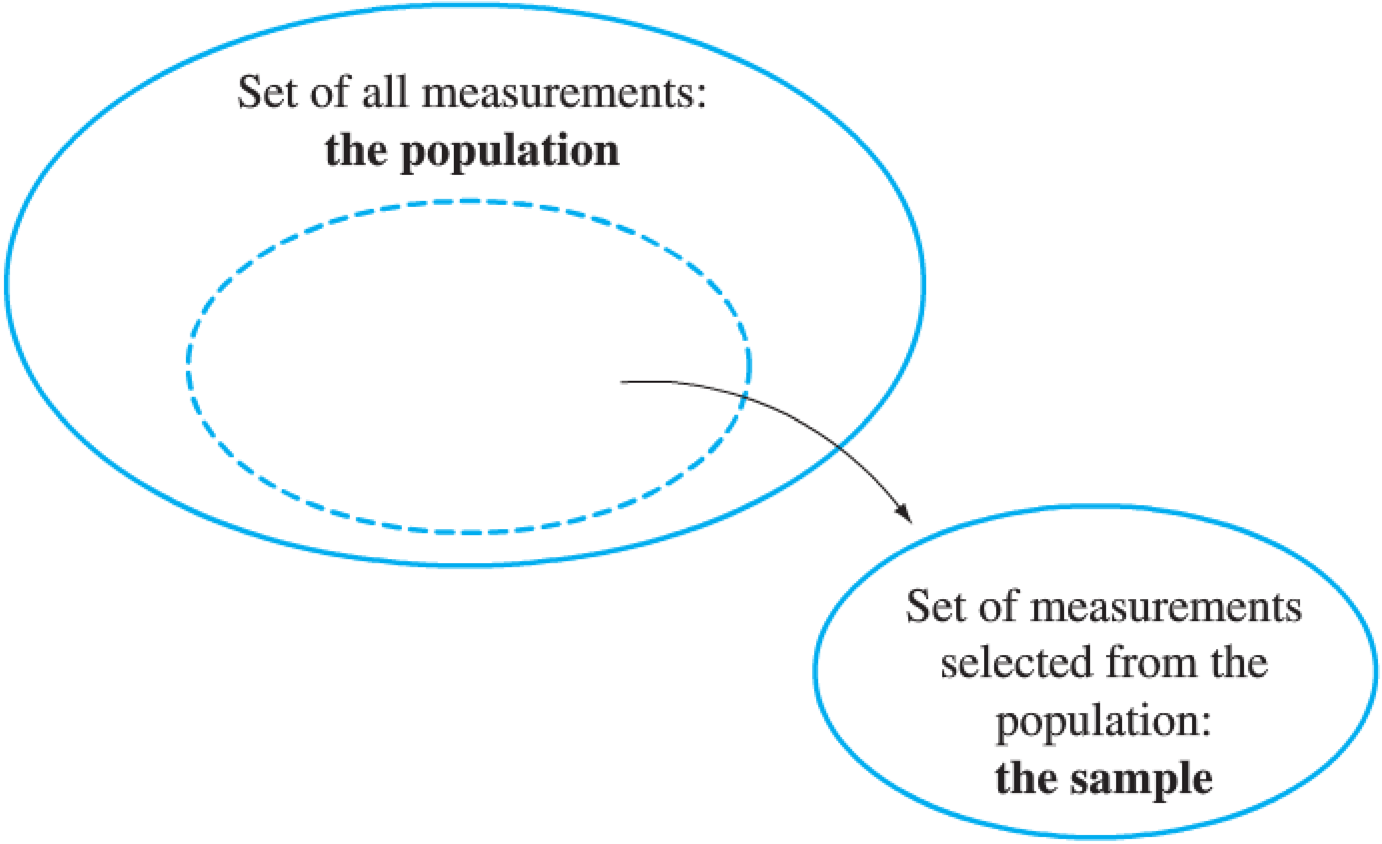
Department of Mathematical and Statistical Sciences

WHAT IS STATISTICS?

- The field of statistics can be divided into two main branches.
- **Descriptive statistics** is what most people think of when they hear the word *statistics*. It includes the collection, presentation, and description of sample data.
- The term **inferential statistics** refers to the technique of interpreting the values resulting from the descriptive techniques and making decisions and drawing conclusions about the population.

POPULATION VS. SAMPLE

- **Population: The entire group of interest**
- **Sample: A part of the population selected to draw conclusions about the entire population**



WHAT IS STATISTICS?

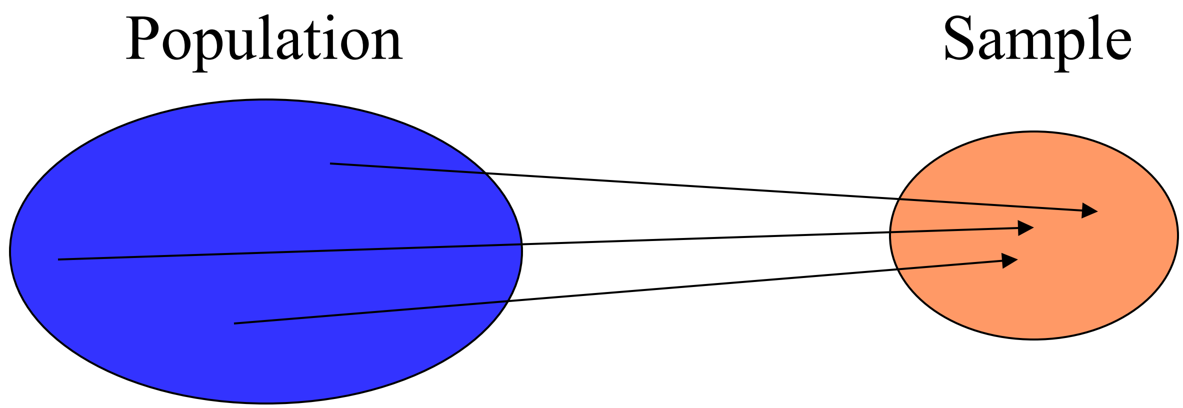
- **Describing a Population**
  - It is common practice to use Greek letters when talking about a population.
  - We call the mean of a population μ.
  - We call the standard deviation of a population σ and the variance σ^2.
  - It is important to know that for a given population there is only **one** true mean and **one** true standard deviation and variance or **one** true proportion.
  - There is a special name for these values: **parameters**.

WHAT IS STATISTICS?

- **Describing a Sample**
  - **We call the mean of a sample $\bar{x}$.**
  - **We call the standard deviation of a sample *s* and the variance *s^2*.**
  - **There are many different possible samples that could be taken from a given population. For each sample there may be a different mean, standard deviation, variance, or proportion.**
  - **There is a special name for these values: statistics.**

POPULATION VS SAMPLE

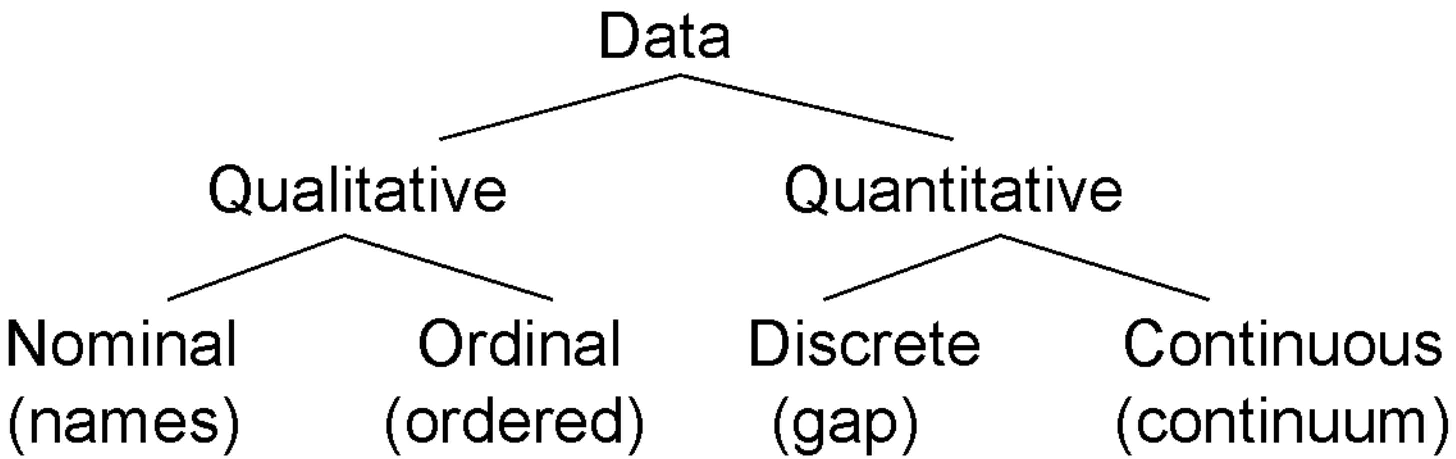
- **We use sample statistics to make inference about population parameters**



Mean:	μ	$\bar{x}$
Standard Deviation:	σ	s

WHAT IS STATISTICS?

- **Data** **The set of values collected from the variable from each of the elements that belong to the sample.**



WHAT IS STATISTICS?

- **Qualitative (Categorical) variable**: A variable that describes or categorizes an element of a population.
  - **Nominal variable**: A qualitative variable that characterizes an element of a population. No ordering. No arithmetic.
  - **Ordinal variable**: A qualitative variable that incorporates an ordered position, or ranking.

- **Quantitative (Numerical) variable**: A variable that quantifies an element of a population.
  - **Discrete variable**: A quantitative variable that can assume a countable number of values. Gap between successive values.
  - **Continuous variable**: A quantitative variable that can assume an uncountable number of values. Continuum of values.

TYPES OF VARIABLES

Examples:

	Numeric		Categorical	
Variable	**Discrete**	**Continuous**	**Nominal**	**Ordinal**
Weight		X		
Hours Enrolled	X			
Major			X	
Zip Code			X	

1. INTRODUCTION: COMBINATIONAL METHODS

THEOREM 1.1. If an operation consists of two steps, of which the first can be done in n_1 ways and for each of these the second can be done in n_2 ways, then the whole operation can be done in $n_1 \cdot n_2$ ways.

THEOREM 1.2. If an operation consists of k steps, of which the first can be done in n_1 ways, for each of these the second step can be done in n_2 ways, for each of the first two the third step can be done in n_3 ways, and so forth, then the whole operation can be done in $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ ways.

THEOREM 1.3. The number of permutations of n distinct objects is $n!$.

THEOREM 1.4. The number of permutations of n distinct objects taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

THEOREM 1.6. The number of permutations of n objects of which n_1 are of one kind, n_2 are of a second kind, $\ldots$, n_k are of a kth kind, and $n_1 + n_2 + \cdots + n_k = n$ is

$$\frac{n!}{n_1! \cdot n_2! \cdot \ldots \cdot n_k!}$$

THEOREM 1.7. The number of combinations of n distinct objects taken r at a time is

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

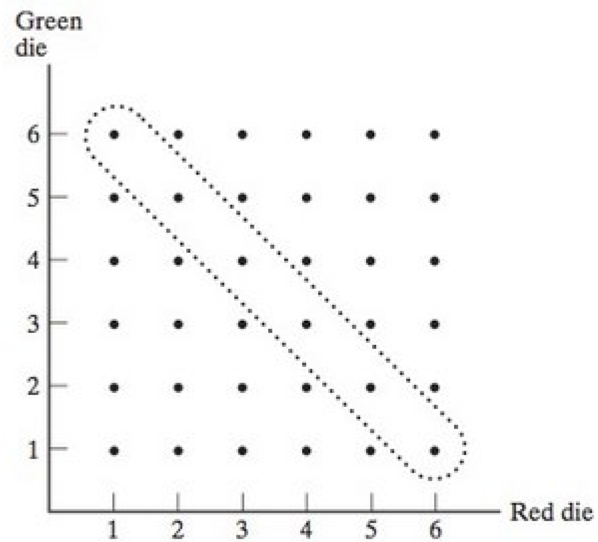
for $r = 0, 1, 2, \ldots, n$.

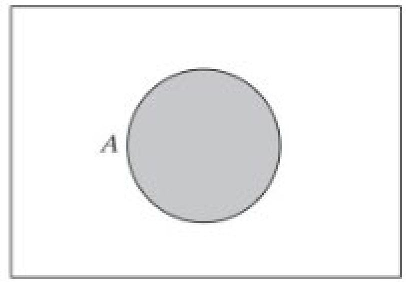
THEOREM 1.9.

$$(x+y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r \quad \text{for any positive integer } n$$

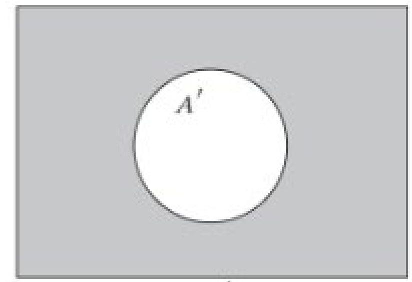
2. PROBABILITY

DEFINITION 2.1. SAMPLE SPACE. *The set of all possible outcomes of an experiment is called the* ***sample space*** *and it is usually denoted by the letter S. Each outcome in a sample space is called an* ***element*** *of the sample space, or simply a* ***sample point****.*

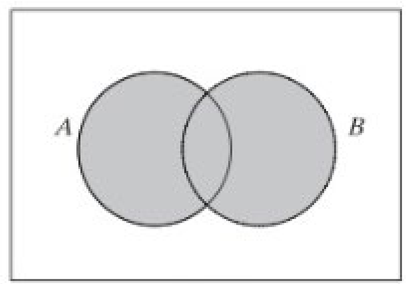




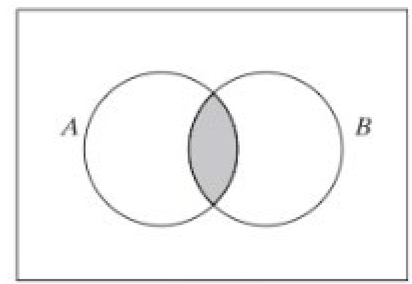
A



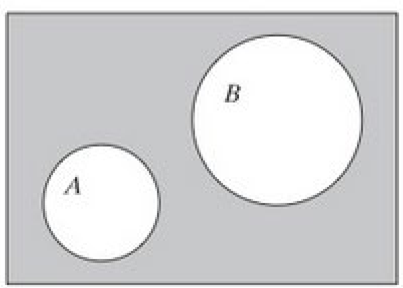
A'



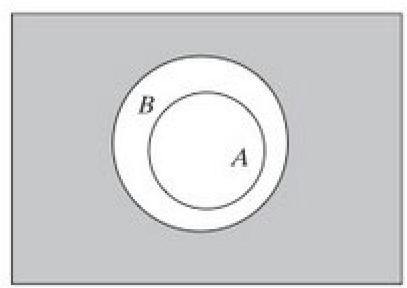
$A \cup B$



$A \cap B$



A and B mutually exclusive



A contained in B

2.4 PROBABILITY OF AN EVENT

POSTULATE 1 The probability of an event is a nonnegative real number; that is, $P(A) \geqq 0$ for any subset A of S.

POSTULATE 2 $P(S) = 1$.

POSTULATE 3 If $A_1, A_2, A_3, \ldots$, is a finite or infinite sequence of mutually exclusive events of S, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

THEOREM 2.3. If A and A' are complementary events in a sample space S, then

$$P(A') = 1 - P(A)$$

THEOREM 2.4. $P(\emptyset) = 0$ for any sample space S.

THEOREM 2.7. If A and B are any two events in a sample space S, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

THEOREM 2.8. If A, B, and C are any three events in a sample space S, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

CONDITIONAL PROBABILITY - INDEPENDENT EVENTS - BAYES RULE

DEFINITION 2.5. INDEPENDENCE. *Two events A and B are **independent** if and only if*

$$P(A \cap B) = P(A) \cdot P(B)$$

DEFINITION 2.4. CONDITIONAL PROBABILITY. *If A and B are any two events in a sample space S and P(A)≠0, the **conditional probability** of B given A is*

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

THEOREM 2.12. If the events $B_1, B_2, \ldots,$ and B_k constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A in S

$$P(A) = \sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)$$

THEOREM 2.13. If $B_1, B_2, \ldots, B_k$ constitute a partition of the sample space S and $P(B_i) \neq 0$ for $i = 1, 2, \ldots, k$, then for any event A in S such that $P(A) \neq 0$

$$P(B_r|A) = \frac{P(B_r) \cdot P(A|B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)}$$

3. PROBABILITY DISTRIBUTIONS AND PROBABILITY DENSITIES

DEFINITION 3.1. RANDOM VARIABLE. *If S is a sample space with a probability measure and X is a real-valued function defined over the elements of S, then X is called a* ***random variable.***†

- **Probability Distributions (Discrete Random Variables)**

DEFINITION 3.2. PROBABILITY DISTRIBUTION. *If X is a discrete random variable, the function given by* $f(x) = P(X = x)$ *for each x within the range of X is called the* ***probability distribution*** *of X.*

THEOREM 3.1. A function can serve as the probability distribution of a discrete random variable X if and only if its values, $f(x)$, satisfy the conditions

1. $f(x) \geqq 0$ for each value within its domain;
2. $\sum_{x} f(x) = 1$, where the summation extends over all the values within its domain.

DEFINITION 3.3. DISTRIBUTION FUNCTION. *If X is a discrete random variable, the function given by*

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t) \qquad \text{for } -\infty < x < \infty$$

where $f(t)$ *is the value of the probability distribution of X at t, is called the* ***distribution function****, or the* ***cumulative distribution*** *of X.*

3.3 CONTINUOUS RANDOM VARIABLES

DEFINITION 3.4. PROBABILITY DENSITY FUNCTION. *A function with values* f(x), *defined over the set of all real numbers, is called a* ***probability density function*** *of the continuous random variable* X *if and only if*

$$P(a \leq X \leq b) = \int_a^b f(x)dx$$

for any real constants a *and* b *with* $a \leq b$.

THEOREM 3.4. If X is a continuous random variable and a and b are real constants with $a \leqq b$, then

$$P(a \leqq X \leqq b) = P(a \leqq X < b) = P(a < X \leqq b) = P(a < X < b)$$

THEOREM 3.5. A function can serve as a probability density of a continuous random variable X if its values, $f(x)$, satisfy the conditions†

1. $f(x) \geqq 0$ for $-\infty < x < \infty$;
2. $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

DEFINITION 3.5. DISTRIBUTION FUNCTION. *If* X *is a continuous random variable and the value of its probability density at* t *is* f(t), *then the function given by*

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt \qquad \text{for } -\infty < x < \infty$$

is called the ***distribution function*** *or the* ***cumulative distribution function*** *of* X.

3.5 MULTIVARIATE DISTRIBUTIONS DISCRETE CASE

DEFINITION 3.6. JOINT PROBABILITY DISTRIBUTION. *If* X *and* Y *are discrete random variables, the function given by* $f(x, y) = P(X = x, Y = y)$ *for each pair of values* (x, y) *within the range of* X *and* Y *is called the* ***joint probability distribution*** *of* X *and* Y.

THEOREM 3.7. A bivariate function can serve as the joint probability distribution of a pair of discrete random variables X and Y if and only if its values, $f(x, y)$, satisfy the conditions

1. $f(x, y) \geqq 0$ for each pair of values (x, y) within its domain;
2. $\sum_x \sum_y f(x, y) = 1$, where the double summation extends over all possible pairs (x, y) within its domain.

DEFINITION 3.7. JOINT DISTRIBUTION FUNCTION. *If* X *and* Y *are discrete random variables, the function given by*

$$F(x, y) = P(X \leq x, Y \leq y) = \sum_{s \leq x} \sum_{t \leq y} f(s, t) \qquad \text{for } -\infty < x < \infty, \; -\infty < y < \infty$$

where $f(s, t)$ *is the value of the joint probability distribution of* X *and* Y *at* (s, t), *is called the* ***joint distribution function***, *or the* ***joint cumulative distribution*** *of* X *and* Y.

3.5 MULTIVARIATE DISTRIBUTIONS CONTINUOUS CASE

MARQUETTE UNIVERSITY
Be The Difference.

DEFINITION 3.8. JOINT PROBABILITY DENSITY FUNCTION. *A bivariate function with values* f(x, y) *defined over the* xy*-plane is called a* ***joint probability density function*** *of the continuous random variables* X *and* Y *if and only if*

$$P(X, Y) \in A = \iint_A f(x, y)dxdy$$

for any region A *in the* xy*-plane.*

THEOREM 3.8. A bivariate function can serve as a joint probability density function of a pair of continuous random variables X and Y if its values, $f(x, y)$, satisfy the conditions

1. $f(x, y) \geqq 0 \qquad \text{for } -\infty < x < \infty, \quad -\infty < y < \infty;$
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1.$

DEFINITION 3.9. JOINT DISTRIBUTION FUNCTION. *If* X *and* Y *are continuous random variables, the function given by*

$$F(x, y) = P(X \le x, Y \le y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(s, t)\, ds\, dt \qquad \text{for } -\infty < x < \infty, \; -\infty < y < \infty$$

where f(s, t) *is the joint probability density of* X *and* Y *at* (s, t)*, is called the* ***joint distribution function of*** X ***and*** Y.

3.6 MARGINAL DISTRIBUTION

DEFINITION 3.10. MARGINAL DISTRIBUTION. *If X and Y are discrete random variables and f(x, y) is the value of their joint probability distribution at (x, y), the function given by*

$$g(x) = \sum_{y} f(x,y)$$

DEFINITION 3.11. MARGINAL DENSITY. *If X and Y are continuous random variables and f(x, y) is the value of their joint probability density at (x, y), the function given by*

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad \text{for } -\infty < x < \infty$$

3.7 Conditional Distribution

DEFINITION 3.12. CONDITIONAL DISTRIBUTION. *If f(x, y) is the value of the joint probability distribution of the discrete random variables X and Y at (x, y) and h(y) is the value of the marginal distribution of Y at y, the function given by*

$$f(x|y) = \frac{f(x,y)}{h(y)} \qquad h(y) \neq 0$$

DEFINITION 3.13. CONDITIONAL DENSITY. *If f(x, y) is the value of the joint density of the continuous random variables X and Y at (x, y) and h(y) is the value of the marginal distribution of Y at y, the function given by*

$$f(x|y) = \frac{f(x,y)}{h(y)} \qquad h(y) \neq 0$$

4. MATHEMATICAL EXPECTATION

DEFINITION 4.1. EXPECTED VALUE. *If X is a discrete random variable and f(x) is the value of its probability distribution at x, the **expected value of X** is*

$$E(X) = \sum_x x \cdot f(x)$$

*Correspondingly, if X is a continuous random variable and f(x) is the value of its probability density at x, the **expected value of X** is*

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

THEOREM 4.1. If X is a discrete random variable and $f(x)$ is the value of its probability distribution at x, the expected value of $g(X)$ is given by

$$E[g(X)] = \sum_x g(x) \cdot f(x)$$

Correspondingly, if X is a continuous random variable and $f(x)$ is the value of its probability density at x, the expected value of $g(X)$ is given by

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) \cdot f(x)\, dx$$

THEOREM 4.2. If a and b are constants, then

$$E(aX + b) = aE(X) + b$$

BIVARIATE CASE

THEOREM 4.4. If X and Y are discrete random variables and $f(x, y)$ is the value of their joint probability distribution at (x, y), the expected value of $g(X, Y)$ is

$$E[g(X,Y)] = \sum_x \sum_y g(x,y) \cdot f(x,y)$$

Correspondingly, if X and Y are continuous random variables and $f(x, y)$ is the value of their joint probability density at (x, y), the expected value of $g(X, Y)$ is

$$E[g(X,Y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f(x,y)\, dx\, dy$$

- **Moments**

DEFINITION 4.2. MOMENTS ABOUT THE ORIGIN. *The* **rth moment about the origin** *of a random variable* X*, denoted by* μ'_r*, is the expected value of* X^r*; symbolically*

$$\mu'_r = E(X^r) = \sum_x x^r \cdot f(x)$$

for $r = 0, 1, 2, \ldots$ *when* X *is discrete, and*

$$\mu'_r = E(X^r) = \int_{-\infty}^{\infty} x^r \cdot f(x) dx$$

when X *is continuous.*

MOMENTS ABOUT THE MEAN

DEFINITION 4.3. MEAN OF A DISTRIBUTION. μ'_1 *is called the* ***mean*** *of the distribution of* X*, or simply the* ***mean of* X***, and it is denoted simply by* μ.

DEFINITION 4.4. MOMENTS ABOUT THE MEAN. *The* ***rth moment about the mean*** *of a random variable* X*, denoted by* μ_r*, is the expected value of* $(X - \mu)^r$*, symbolically*

$$\mu_r = E\left[(X-\mu)^r\right] = \sum_x (x-\mu)^r \cdot f(x)$$

for $r = 0, 1, 2, \ldots$*, when* X *is discrete, and*

$$\mu_r = E\left[(X-\mu)^r\right] = \int_{-\infty}^{\infty} (x-u)^r \cdot f(x)dx$$

when X *is continuous.*

DEFINITION 4.5. VARIANCE. μ_2 *is called the* ***variance*** *of the distribution of* X*, or simply the* ***variance of* X***, and it is denoted by* σ^2, $\sigma_X{}^2$, *var*(X)*, or* V(X)*. The positive square root of the variance,* σ*, is called the* ***standard deviation of* X**.

THEOREM 4.6.

$$\sigma^2 = \mu'_2 - \mu^2$$

THEOREM 4.7. If X has the variance σ^2, then

$$\text{var}(aX+b) = a^2\sigma^2$$

4.4 CHEBYSHEV'S THEOREM

THEOREM 4.8. (*Chebyshev's Theorem*) If μ and σ are the mean and the standard deviation of a random variable X, then for any positive constant k the probability is *at least* $1 - \frac{1}{k^2}$ that X will take on a value within k standard deviations of the mean; symbolically,

$$P(|x - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}, \quad \sigma \neq 0$$

- **Moment Generating Functions**

DEFINITION 4.6. MOMENT GENERATING FUNCTION. *The* ***moment generating function*** *of a random variable* X*, where it exists, is given by*

$$M_X(t) = E(e^{tX}) = \sum_x e^{tX} \cdot f(x)$$

when X *is discrete, and*

$$M_X(t) = E(e^{tX}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x)dx$$

when X *is continuous.*

THEOREM 4.9.

$$\left.\frac{d^r M_X(t)}{dt^r}\right|_{t=0} = \mu_r'$$

INDEPENDENT RANDOM VARIABLES

DEFINITION 3.14. INDEPENDENCE OF DISCRETE RANDOM VARIABLES. *If* $f(x_1, x_2, \ldots, x_n)$ *is the value of the joint probability distribution of the discrete random variables* $X_1, X_2, \ldots, X_n$ *at* $(x_1, x_2, \ldots, x_n)$ *and* $f_i(x_i)$ *is the value of the marginal distribution of* X_i *at* x_i *for* $i = 1, 2, \ldots, n$, *then the* n *random variables are* ***independent*** *if and only if*

$$f(x_1, x_2, \ldots, x_n) = f_1(x_1) \cdot f_2(x_2) \cdot \ldots \cdot f_n(x_n)$$

for all $(x_1, x_2, \ldots, x_n)$ *within their range.*

THEOREM 4.12. If X and Y are independent, then $E(XY) = E(X) \cdot E(Y)$ and $\sigma_{XY} = 0$.

THEOREM 4.13. If $X_1, X_2, \ldots, X_n$ are independent, then

$$E(X_1 X_2 \cdot \ldots \cdot X_n) = E(X_1) \cdot E(X_2) \cdot \ldots \cdot E(X_n)$$

THEOREM 4.14. If $X_1, X_2, \ldots, X_n$ are random variables and

$$Y = \sum_{i=1}^{n} a_i X_i$$

where $a_1, a_2, \ldots, a_n$ are constants, then

$$E(Y) = \sum_{i=1}^{n} a_i E(X_i)$$

and

$$\text{var}(Y) = \sum_{i=1}^{n} a_i^2 \cdot \text{var}(X_i) + 2 \sum\sum_{i<j} a_i a_j \cdot \text{cov}(X_i X_j)$$

COVARIANCE(X,Y)

- $\text{var}[aX \pm bY] = a^2\text{var}[X] + b^2\text{var}[Y] \pm 2ab\ \text{cov}[X,Y]$,

where $\text{cov}[X,Y] = E\{(X - E[X])(Y - E[Y])\}$
$= E[X.Y] - E[X].E[Y]$

- **Conditional Expectation**

DEFINITION 4.10. CONDITIONAL EXPECTATION. *If* X *is a discrete random variable, and* f(x|y) *is the value of the conditional probability distribution of* X *given* Y = y *at* x, *the* ***conditional expectation of* u(X) *given* Y = y** *is*

$$E[u(X)|y)] = \sum_x u(x) \cdot f(x|y)$$

Correspondingly, if X *is a continuous variable and* f(x|y) *is the value of the conditional probability distribution of* X *given* Y = y *at* x, *the* ***conditional expectation of* u(X) *given* Y = y** *is*

$$E[(u(X)|y)] = \int_{-\infty}^{\infty} u(x) \cdot f(x|y)dx$$

CHAPTER 5.
COMMON DISCRETE DISTRIBUTIONS:

- **Discrete Uniform (Wiki)**

- **Bernouli(Wiki)**
- **Binomial (Wiki)**
- **Negative Binomial(Wiki)**
- **Geometric(Wiki)**
- **Hypergeometric(Wiki)**
- **Poisson (Wiki)**
- **Multinomial(Wiki)**
- **Multivariate Hypergeometric Distribution**

CHAPTER 6.
COMMON CONTINUOUS DISTRIBUTIONS:

- Uniform
  - Wiki
- Exponential
  - Wiki
- Gamma
  - Wiki
- Beta
  - Wiki
- Weibull
  - Wiki
- Cauchy
  - Wiki
- Normal (μ=mean, σ^2=variance)
  - Wiki
- Bivariate Normal (Wiki)
- Skew Normal (Wiki)
- t (ν=df)
  - Wiki
- Chi-Square (ν=df)
  - Wiki
- F (ν_1=df$_1$, ν_2=df$_2$)
  - Wiki

- **7.2 Distribution Function Technique**

A straightforward method of obtaining the probability density of a function of continuous random variables consists of first finding its distribution function and then its probability density by differentiation. Thus, if $X_1, X_2, \ldots, X_n$ are continuous random variables with a given joint probability density, the probability density of $Y = u(X_1, X_2, \ldots, X_n)$ is obtained by first determining an expression for the probability

$$F(y) = P(Y \leqq y) = P[u(X_1, X_2, \ldots, X_n) \leqq y]$$

- **7.3 Transformation Technique: One Variable**

THEOREM 7.1. Let $f(x)$ be the value of the probability density of the continuous random variable X at x. If the function given by $y = u(x)$ is differentiable and either increasing or decreasing for all values within the range of X for which $f(x) \neq 0$, then, for these values of x, the equation $y = u(x)$ can be uniquely solved for x to give $x = w(y)$, and for the corresponding values of y the probability density of $Y = u(X)$ is given by

$$g(y) = f[w(y)] \cdot |w'(y)| \quad \text{provided } u'(x) \neq 0$$

Elsewhere, $g(y) = 0$.

7.4 TRANSFORMATION TECHNIQUE: SEVERAL VARIABLES

Theorem 7.1 with the transformation formula written as

$$g(y, x_2) = f(x_1, x_2) \cdot \left| \frac{\partial x_1}{\partial y} \right|$$

or as

$$g(x_1, y) = f(x_1, x_2) \cdot \left| \frac{\partial x_2}{\partial y} \right|$$

where $f(x_1, x_2)$ and the partial derivative must be expressed in terms of y and x_2 or x_1 and y. Then we integrate out the other variable to get the marginal density of Y.

THEOREM 7.2. Let $f(x_1, x_2)$ be the value of the joint probability density of the continuous random variables X_1 and X_2 at (x_1, x_2). If the functions given by $y_1 = u_1(x_1, x_2)$ and $y_2 = u_2(x_1, x_2)$ are partially differentiable with respect to both x_1 and x_2 and represent a one-to-one transformation for all values within the range of X_1 and X_2 for which $f(x_1, x_2) \neq 0$, then, for these values of x_1 and x_2, the equations $y_1 = u_1(x_1, x_2)$ and $y_2 = u_2(x_1, x_2)$ can be uniquely solved for x_1 and x_2 to give $x_1 = w_1(y_1, y_2)$ and $x_2 = w_2(y_1, y_2)$, and for the corresponding values of y_1 and y_2, the joint probability density of $Y_1 = u_1(X_1, X_2)$ and $Y_2 = u_2(X_1, X_2)$ is given by

$$g(y_1, y_2) = f[w_1(y_1, y_2), w_2(y_1, y_2)] \cdot |J|$$

Here, J, called the **Jacobian** of the transformation, is the determinant

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix}$$

Elsewhere, $g(y_1, y_2) = 0$.

7.5 MOMENT-GENERATING FUNCTION TECHNIQUE

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THEOREM 7.3. If $X_1, X_2, \ldots$, and X_n are independent random variables and $Y = X_1 + X_2 + \cdots + X_n$, then

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

where $M_{X_i}(t)$ is the value of the moment-generating function of X_i at t.

- **ANY QUESTION?**